[3]Huang, C. L., "On Strength Function for Anisotropic Materials," *Proceedings of Symmetry, Similarity and Group-Theoretic Methods in Mechanics*, The University of Calgary, Canada, 1974.
[4]Huang, C. L. and Kirmser, P. G., "A Criterion on Strength for Orthotropic Materials," *Fiber Science and Technology*, 1975, to be published.
[5]Ely, R. E., "Strength Results for Two Brittle Materials Under Biaxial Stresses," U. S. Army Missile Command—Redstone Arsenal Alabama, Technical Rept. RR-72-11, 1972.
[6]Weng, T. L., "Grade ATJ Graphite," *Proceedings of the Conference on Continuum Aspects of Graphite Design*, Nov. 9-12, Gatlinburg, Tennessee, CONF-701105, NTIS, U.S. Dept. of Commerce, 1970.
[7]Hoffman, O., "The Brittle Strength of Orthotropic Materials," *Journal of Composite Materials*, Vol. 1, 1967, pp. 200-206.
[8]Hill, R., "A Theory of the Yielding and Plastic Flow of Anisotropic Metals," Procedings Royal Society (London), A, 193, 1948, pp. 281-297.

Calculation of Compressible Turbulent Boundary Layers on Straight-Tapered Swept Wings

P. Bradshaw*
Imperial College, London, England
G. A. Mizner†
Queen Mary College, London, England
and
K. Unsworth‡
Imperial College, London, England

Introduction

THE turbulent boundary layer on a constant-chord swept wing (or infinite yawed wing) is aerodynamically three-dimensional, because the velocity and shear-stress vectors do not coincide in plan view; but it is computationally two-dimensional, because derivatives along the generators are zero.[1-4] Infinite yawed wings are a useful approximation to real wings, but it does not seem to have been realized that calculations for straight-tapered wings (a very good approximation to many real wings) are only slightly more complicated. Here we describe the extension of an infinite-wing program to the case of a straight-tapered wing. The program calculates compressible flow with or without heat transfer.

Over the outer, straight-tapered part of the wing shown in Fig. 1, derivatives of external-flow quantities along the generators (spanwise) are nearly zero except near the tip. Because the boundary-layer thickness varies along the generators, spanwise derivatives within the boundary layer are not zero, but they can easily be related to derivatives normal to the surface, so that, as on an infinite yawed wing, calculations are needed at only one spanwise station, from which results at other stations follow by scaling with the local chord. The full equations for nonaxisymmetric flow in cylindrical polar coordinates are given by Rodi:[5] his x, r, θ correspond to the present y, $r_0 - z$, x/r_0, respectively. All equations contain extra terms which can be loosely described as rotation-of-axes terms. The Reynolds-stress transport equations contain extra generation terms which are individually of the same order as the rotation-of-axes terms but whose net effect is much smaller. Nash and Scruggs[6] used a cylindrical polar coordinate system in calculations of the turbulent boundary layer on the straight-tapered wing shown in Fig. 1; they neglected the extra generation terms without explanation, and did not notice that derivatives along the generators could be simplified.

Received September 17, 1975. This work was supported by Ministry of Defence (PE) under contract AT/2037/0133: helpful discussions with the project monitor, L. F. East, are gratefully acknowledged.
Index category: Boundary Layers and Convective Heat Transfer—Turbulent.
*Reader in Fluid Dynamics.
†Research Student.
‡Postdoctoral Research Assistant.

Table 1 Calculations of Nash and Scruggs[6] for the swept wing of Fig. 1 at $M_\infty = 0.5$. C_F = chordwise-average skin-friction coefficient

Station	A	B	C	D	E
$10^3 \delta^*_{TE}/c$	2.49	2.75	2.71	2.72	3.62
$10^3 C_F$	3.40	3.50	3.54	3.62	3.62

Nash's and Scruggs' predictions of the skin friction coefficient, and of the displacement thickness divided by local chord at the trailing edge of the wing shown in Fig. 1 are given in Table 1. The variation along the span is negligible, as our straight-taper approximation would predict, except near the crank and the tip; a small Reynolds-number effect on skin friction can be seen. The report from which Nash and Scruggs took the pressure distributions is not publicly available so we have not been able to check the present method against their calculations (which used a slightly simplified version of the present turbulence model). However their finding that the isobars on a real straight-tapered wing conicide quite closely with the generators, even at $M_\infty = 0.99$, justifies the approach in the present paper.

The Program

We have programed the compressible heat transfer version[7] of the boundary-layer calculation method of Bradshaw and Ferriss,[8] for straight-tapered wings. The extension of the incompressible isothermal version to infinite swept wings was described in Ref. 2 and the present version is essentially the same aerodynamically. The two components of the Reynolds shear stress τ, $-\overline{\rho u v}$ and $-\overline{\rho v w}$, are predicted by transport equations, which are a logical extension to three dimensions of the empirical shear stress transport equation derived from the turbulent energy equation in Ref. 8. It should be noted that this shear-stress equation is conceptually equivalent to those derived by later workers from the exact shear-stress transport equation; it has the advantage of being based on an exact equation whose main terms have been measured. The heat-transfer calculation uses a slightly refined version of the usual assumption of constant Prandtl number. In contrast to integral methods, crossover profiles present no difficulty. The main advantage over the integral methods actually available for three-dimensional flows is the use of a modern turbulence

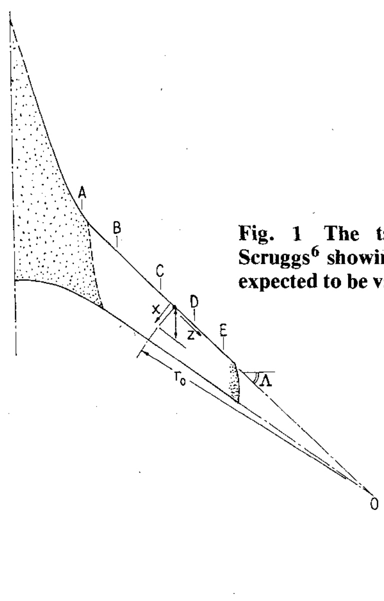

Fig. 1 The tapered wing of Nash and Scruggs[6] showing the present axes. Method is expected to be valid outside dotted regions.

model. The program runs more slowly, but the present tapered-wing version should certainly be competitive with integral methods needing several spanwise stations.

To a good approximation (neglecting the effect of the spanwise variation of Reynolds number based on chord) δ at given x is proportional (Fig. 1) to $r_0 - z$ and, for any function f, $\partial f/\partial z$ at constant y is equal to $y/r_0\ \partial f/\partial y$. Thus in the transport equation for f, $V\partial f/\partial y$ and $W\partial f/\partial z$ combine to yield $(V+Wy/r_0)\partial f/\partial y$. As in Ref. 9, $\bar{\rho} V$ is augmented by the turbulent mass flux in the y direction, $\overline{\rho' v}$. If we denote $V+\overline{\rho' v}/\rho + Wy/r_0$ by $\hat{V}$ the equations for u, $\hat{V}$, W, $-\rho\overline{uv}$ and $-\rho\overline{vw}$ are the same as for an infinite wing except for the rotation of axes terms.

The program contains a large number of options for input, output, and physical effects, selected individually by choosing nonzero values of integer control parameters so that default operation is obtained with two essential control parameters and a row of blanks.[7] The number of y profile points must be chosen by the user, but in general the program is no more complicated to use than an integral method. Aerodynamic options include surface roughness[11] and allowances for the effects of surface curvature,[12] lateral convergence/divergence, bulk dilatation[13] and freestream turbulence on the turbulence structure. The program contains just under 1000 Fortran statements, executes in 8,500 (decimal) words plus computer library space and marches in the x direction at about 8 boundary-layer thicknesses per second on a CDC 6400 (this is about two-thirds the speed of the two-dimensional compressible heat transfer program or half the speed of the incompressible isothermal program).

The empirical data used by the program are all obtained from two-dimensional flow; with the exception of the allowance for bulk dilatation effects and the effect of Mach number on the additive constants in the logarithmic inner-layer profiles, the data all come from incompressible flow and those of Ref. 9 and 13 in two-dimensional compressible flow, while in the later case, the Reynolds analogy factor in constant-pressure flow is close to 1.16. The results in zero pressure gradient are acceptable up to a freestream Mach number M_e of at least 10 although the Morkovin hypothesis[14] justifying the turbulence modeling requires that the local Mach number M shall satisfy $(\gamma-1)M^2 \not\gg 1$. The Reynolds analogy factor in constant-pressure flow remains close to 1.16 over the range $0.25 < T_w/T_{oe} < 4$. There is no explicit limit on sweep angle, but the assumption of radial isobars and the neglect of influence spreading through the boundary layer from other parts of the wing are likely to fail in many cases where the sweep angle exceeds, say, 45° and are certain to fail in most cases near the root and tip of the wing. It is likely that the assumption (which can be checked) will in practice fail before the neglect (which cannot). The program will run to within one x step of separation: van den Berg et al.[15] have recently found discrepancies between existing calculation methods (including the present one) and their measurements of a separating boundary layer on an infinite yawed wing, but it is not clear that the descrepancies are connected with the three-dimensionality.

Fortran card decks are available from the first author for the cost of reproducing and mailing. Subsets of the full program, e.g., two-dimensional compressible heat transfer, are also available.

References

[1] Thompson, B. G. J., Carr-Hill, G. A., and Ralph, M., "The Prediction of Boundary-Layer Behavior and Profile Drag for Infinite Yawed Wings. Part III," CP 1309, 1973, Aeronautical Research Council, London, England.

[2] Bradshaw, P., "Calculation of Three-Dimensional Turbulent Boundary Layers," *Journal of Fluid Mechanics*, Vol. 46, 1971, p. 417.

[3] Cebeci, T., Mosinskis, G. J., and Kaups, K., "A General Method for Calculating Three-Dimensional Incompressible Laminar and Turbulent Boundary Layers. I," Douglas Aircraft Co., MDC J5694, 1972.

[4] Adams, J. C., "Calculation of Compressible Turbulent Boundary Layers on an Infinite Yawed Aerofoil," *Journal of Spacecraft*, Vol. 12, 1975, p. 131.

[5] Rodi, W., "Basic Equations for Turbulent Flow in Cartesian and Polar Coordinates," Imperial College, BL/TN/A/36, 1970, and Corrigenda, 1971.

[6] Nash, J. F. and Scruggs, R. M., "Three Dimensional Compressible Boundary Layer Computations for a Finite Swept Wing," CR-112158, 1972, NASA.

[7] Bradshaw, P. and Unsworth, K., "An Improved Fortran Program for the Bradshaw-Ferriss-Atwell Method of Calculating Turbulent Shear Layers," Aero Rept. 74-02, 1974, Imperial College, London.

[8] Bradshaw, P. and Ferriss, D. H., "Applications of a General Method of Calculating Turbulent Shear Layers," *Transactions of the ASME Basic Engineering*, Vol. 94D, 1972, p. 345.

[9] Bradshaw, P. and Ferriss, D. H., "Calculation of Boundary Layer Development Using the Turbulent Energy Equation. Compressible Flow on Adiabatic Walls," *Journal of Fluid Mechanics*, Vol. 46, 1971, p. 83.

[10] Bradshaw, P., Mizner, G. A., and Unsworth, K., "Calculation of Compressible Turbulent Boundary Layers with Heat Transfer on Straight-Tapered Swept Wings," Aero Rept. 75-04, 1975, Imperial College, London.

[11] Goddard, F. E., "Effect of Uniformly-Distributed Roughness on Turbulent Skin-Friction Drag at Supersonic Speeds," *Journal of Aerospace Science*, Vol. 26, 1959, p. 1.

[12] Bradshaw, P., "Effect of Streamline Curvature on Turbulent Flow," AGARDograph 169, 1973.

[13] Bradshaw, P., "The Effect of Mean Compression or Dilatation on the Turbulence Structure of Supersonic Boundary Layers," *Journal of Fluid Mechanics*, Vol. 63, 1974, p. 449.

[14] Morkovin, M. V., "Effects of Compressibility on Turbulent Flows," *The Mechanics of Turbulence*, A. Favre, ed., Gordon & Breach, New York, 1964, p. 367.

[15] Van den Berg, B.., Elsenaar, A., Lindhout, J., and Wesseling, P., "Measurements in an Incompressible Three-Dimensional Turbulent Boundary Layer, under Infinite Swept Wing Configurations, and Comparison with Theory," *Journal of Fluid Mechanics*, Vol. 70, 1975, p. 127.

Effect of Velocity Gradients on Measurements of Turbulent Shear Stress

V.A. Sandborn*

Colorado State University, Fort Collins, Colo.

Introduction

IN a recent Note Johnson and Rose[1] demonstrated that both laser velocimeter and hot wire measurements obtained in a supersonic boundary layer fail to give the expected values of the turbulent shear stress near the surface. In the outer portion of the boundary layer both instruments agree approximately with the general flat plate similarity shear distribution proposed by Sandborn.[2] While a question on the use of $\rho\overline{uv}$ as the total shear stress was posed by Johnson and Rose, it is likely that a probe measurement difficulty exists near the surface.

Measurements in regions of large mean and turbulent velocity gradients can produce correspondingly large errors. For the boundary layer studied by Johnson and Rose, the thickness was of the order of 2.5cm, with the mean velocity varying from zero to 550 m/sec over this short distance. The

Received September 29, 1975; revision received December 1, 1975. The author gratefully acknowledges the support of the NASA-Ames Research Center in the research.

Index category: Boundary Layer and Convective Heat Transfer-Turbulent.

*Professor of Civil Engineering. Member AIAA.